Internationalizing the University Mathematics Curriculum

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Abstract. In this paper we examine issues that arise in internationalizing the university mathematics curriculum and offer suggestions for achieving this internationalization.

Keywords: internationalizing curriculum, ethnomathematics, cross-national studies, mathematics education.

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Internationalizing the University Mathematics Curriculum

Section 1: Introduction.

In this paper we offer suggestions for achieving the internationalization of university mathematics-related curricular offerings. We present learning objectives, raise general issues related to internationalization, and then discuss how to incorporate the objectives and address these issues within the university mathematics curriculum. Opinions on the issues we raise in this paper are often divided between those who are professional mathematicians and those who are in mathematics education; between those who believe that mathematics is already inherently international, and those who believe that there are cultural aspects which affect the teaching of mathematics. This group of authors has had many such discussions and, while we still do not have a complete consensus, we have all come to believe that it is important that mathematics students become aware of the cultural and political implications of internationalization. Our approach is to formulate pragmatic learning outcomes whereby students are given opportunities to grapple with the important questions of internationalization in order to formulate their own positions. Our goal is not to impose our own opinions on the students but rather to have them engage meaningfully in the dialogue.

We are following the lead of the American Council on Education (ACE) in using the term "internationalization" to denote the incorporation of an international/intercultural dimension in teaching (Siaya and Hayward, 2003, p. xi). This definition is subsumed by the more comprehensive one proposed by Knight (2003): "Internationalization at the national, sector, and institutional levels is defined as the process of integrating an international, intercultural, or global dimension into the purpose, functions or delivery of postsecondary education."

Some authors prefer the term "globalization." However, as Atweh and Clarkson (2001) note, it is not universally agreed that the two terms are equivalent. According to David Rothkopf, quoted in Friedman (2005, page 45), the word "globalization" was originally coined by the Clinton administration (in which Rothkopf served as a senior Department of Commerce official) to describe the changing relationships between big governments and big businesses but has since evolved to include "the emergence of new social, political, and business models." Knight acknowledges the "complex and rather contentious topic of globalization" and offers the following non-ideological definition of the term: "the flow of technology, economy, knowledge, people, values and ideas across borders." She sees globalization as a multifaceted phenomenon that impacts the internationalization of education but believes that it should not be at the center of discussion on internationalization. Thus she argues against the use of the term "globalization of education."

The importance of internationalizing college and university curricula is widely accepted. According to Green and Shoenberg (2006), "Internationalizing the curriculum is the most important strategy institutions can use to ensure that *all* their students acquire the knowledge, skills, and attitudes they will need as citizens and workers in a rapidly changing and globalized world." Institutions of higher education have tried to respond to this call. “It would be difficult to find a college or university today that is not making some effort to internationalize.”(2006). While a facile response would involve relegating the international topics to a few general education requirements, there is an increasing awareness that to be most effective internationalization should be incorporated into all majors and programs. Indeed, realizing that true internationalization would have to be led by individual departments and disciplines, ACE (Green & Shoenberg, 2006) conducted a joint project with the professional organizations of four disciplines (Geography, History, Political Science, and Psychology) so that each could formulate its own set of specific global learning outcomes. ACE recognized that some disciplines are more easily internationalized than others, and it is interesting to note that all four of the disciplines chosen for the project are from the humanities and social sciences.

The challenge of internationalizing the university mathematics curriculum is reflected in the relative paucity of papers on this subject found in the literature. This is ironic since in some ways mathematics is among the most international and collaborative of all academic disciplines. As Atweh and Clarkson (2002A, p. 11) point out, “the subject itself is a product of centuries of cultural exchange between East and West.” Furthermore, as discussed in section 3 of this paper, the proportion of papers co-authored by researchers of different nationalities is higher in mathematics than in nearly every other discipline. However, a key question to have students consider is whether mathematics, while developed through international efforts, has “achieved a status of an international language independent of cultural affiliation and context of development” (Atweh and Clarkson, 2002A, p.10.) This view would clearly differentiate mathematics from many of the fields in the humanities and social sciences which have components that are clearly unique to each country and culture.

Some of the strategies that we suggest for internationalizing the curriculum involve the humanities and social science aspects of mathematics. These recommendations include infusing the History of Mathematics course with more non-Western content and developing a new inter-disciplinary course to be jointly offered by the mathematics and sociology/anthropology departments. Since these courses will not be part of a traditional mathematics major, we put forth other suggestions to address how these topics can impact students. These suggestions include the development of a new course in mathematical modeling, enhancements to the major capstone course, and a commitment to increasing the proportion of our students who study abroad. We continue to explore additional ways of meeting the ultimate goal of infusing international perspectives into all our existing courses.

Internationalizing the mathematics curriculum involves learning about and being sensitive to cultural differences throughout the world and that in turn has lead the authors to consider multi-cultural issues within our own country. Thus, some of our proposals address both types of learning. Our emphasis, however, is on internationalization.

Section 2: Global Learning Objectives for Undergraduate Mathematics

1. Students should recognize the contributions and parallel developments of mathematical concepts in non-western cultures.
    a. Outcome: Students will be able to place the history of mathematical ideas in global contexts, including the achievements of non-Western cultures.
    b. Outcome: Students will be able to describe several critical examples of how students of mathematics from different cultures may approach mathematics differently.
    c. Outcome: Students will be able to explain the key features of common arguments that mathematics as a field is Western-dominated and will be able to apply these arguments to formulate their own position on the issue.
2. Students should be aware of the social role that mathematics can play across different cultures.
    a. Outcome: Students will be able to illustrate how mathematics can impact the framing of social issues and formulate a personal position on this impact.
    b. Outcome: Students will be able to describe how mathematics may play different roles in different cultures.
3. Students will have first-hand experience interacting with creators and users of mathematics from other countries and cultures.
    a. Outcome: Students will be encouraged to study abroad or to work in a community that is different from that in which they themselves were raised and to reflect on how the experience has or has not changed their views of mathematics as a cultural practice.
4. Students will be able to view the role that mathematics plays in modeling and solving problems from an international perspective.
    a. Outcome: Students will illustrate with several examples how social norms and taboos can influence the interpretation of mathematical models in different cultures.
    b. Outcome: Students will choose appropriate mathematical models to describe a variety of social phenomena, explain their choice, and describe the limitations of the models they have selected.

Section 3: Issues

In this section we consider several issues in the literature which are directly related to the internationalization of the university mathematics curriculum. Not all of these issues correspond to single objectives as listed above; some relate to several objectives, while others increase our understanding of larger questions related to internationalization.

A. Ethnomathematics.

Ethnomathematics presents the idea that an indigenous mathematics often arises in many contexts and in many cultures. The original concept and terminology were developed by the Brazilian mathematician Ubiratan D'Ambrosio in the early 1980's. (Ascher & Ascher, 1986; D'Ambrosio, 1989). This concept closely corresponds to the first learning objective of Section 2: "Students should recognize the contributions and parallel developments of mathematical concepts in non-western cultures." Many mathematics educators now include these ideas in their teaching as a method to reach out to students of different backgrounds. For example, Professor Jim Barta of Utah State University reports that he has observed American Indian students change their perception of mathematics when they recognize its place in their culture (Riley, 2006). He goes on to say that math without context or culture becomes meaningless and inhibits learning. Eugene Klotz, Founder of the Math Forum, has commented that he uses ideas of Ethnomathematics to attract students to mathematics (Klotz, 2007). There are many rich resources including the books by Ascher (2003) and Powell and Frankenstein (1997) as well as the webpages of Ron Eglash (2007). Another reason that both students and teachers should be encouraged to study ethnomathematics is so that they are more sensitive to cultural differences and learning styles in the classroom. (L. Friedler, 1998).

Despite the increasing popularity of these ideas, the role of ethnomathematics in the teaching of mathematics has become a focus of debate. There are proponents who look to ethnomathematics as an approach that raises a critical perspective on the discipline and professional practice of mathematics internationally. This idea is based on two conflicting tenets. On one hand, mathematics is central to Western notions of knowledge and rationality (Walkerdine 1987; Appelbaum 1995). Because of this, mathematics is often held up as a symbol of colonialism and imperialism and blamed for perpetuating international power dynamics. Yet the profession of mathematics is fairly consistently understood internationally by mathematicians to be the very same inherited set of questions, techniques, conventions and practices. There is a very real acceptance of the primarily Western conception of mathematics as a scholarly discipline that all students, regardless of nationality, should be trained to enter (Jablonka, 2003). Bishop (1991) and (independently) Nunes (1992) have suggested that the same set of questions arise independently in different cultures. Some might imagine, along with Alan Bishop (2001), a mathematics curriculum within a cultural perspective that does not privilege one tradition over another, building from Bishop's cultural universals of counting, locating, measuring, designing, playing and explaining.

On the other hand, there are scholars who look at the role of ethnomathematics as limited to the setting of the teaching of the history of mathematics. Thus, for example, Marcia Asher, in her book *Ethnomathematics: A Multicultural View of Mathematical Ideas* (Asher, 2003) advocates broadening the teaching of the history of mathematics to include the achievements of other cultures and of people who are not necessarily recognized as professional mathematicians. (See also Fasheh, 1990)

Finally, some scholars look in distrust at both of the above approaches. For many, as Judith Grabiner points out in her review of Asher's book (Grabiner, 1993), "Ethnomathematics is not the same as history of mathematics." She goes on to suggest that good histories of mathematics have always included contributions of non-Western cultures. Yet, "the mathematics that is taught in universities is the mathematics whose history we want to study." Some mathematicians go further. Chartrand and Zhang (2005) state in the preface to their text, "In the ancient past, certain cultures developed their own mathematics. In recent centuries, there has become only one international mathematics."

We believe university students should encounter questions about the role of ethnomathematics in the university mathematics curriculum. Some particular questions they should address include: Should we use ethnomathematics only as a source of examples to illustrate mathematical concepts? Should ethnomathematics be the broader context within which all mathematics is taught and learned or is it primarily of historical importance? Is there, in fact, a distinct body of knowledge recognized as Western mathematics?

From the point of view of the internationalization of the university mathematics curriculum, it is our belief that ideas from ethnomathematics have an important role to play. The authors of this paper have discovered that despite having different positions in the above-mentioned debate, there are some curricular innovations that we all agree will be beneficial in increasing the internationalization of the mathematics curriculum. (See Section 4 for details of our proposals). Our consensus is based on two observations. First, one must avoid the trap of approaching ethnomathematics as a catalog of examples to be decontextualized and reduced to pure mathematical problems. By failing to note the role of culture in the scientific achievements of a civilization, this approach is essentially the "producing of catalogs at the expenses of context" (Meier and Thorne 1999). Second, we believe that although it is important to be aware of the effect of culture, we cannot lose sight of the mathematical concepts in our curriculum that the wider mathematical community believes are central. In Section 3 we will return to examples of treating mathematics and their cultural contexts equally; these examples will appear in our proposed modeling and cultural contexts courses.

B. Uniformity of elementary and secondary mathematics curriculum.
Is globalization leading to a uniformity of the elementary and secondary mathematics curriculum? (In the U.S., this is commonly called the K-12, or Kindergarden-12th grade, curriculum.) There is increasing concern that the mathematics curriculum has not only become globalized but Westernized. (Atweh & Clarkson, 2002). This issue is directly related to Western domination of mathematics addressed in Outcome 1c and naturally leads to a discussion of how different cultures approach mathematics as in Outcome 1b. The NCTM report of 1989 led many countries to adopt curriculum standards. Yet whose standards were adopted? When Hong Kong revised its secondary math curriculum in 2000, great care was taken to insure that the curriculum was in line with world trends (Wong, Han & Lee, 2004). Further, many authors (Atweh & Carlson, 2001) have pointed to the similarities of mathematics textbooks around the world. Many justify a uniform curriculum by the need for graduates to be able to compete in global markets. Atweh and Carlson (2002) point to colonialization as a source of this uniformity, so that "a number of the colonialized countries have modeled their educational systems… on that of the mandate countries." (p. 160) Also, many students study abroad and "return to their home countries to occupy prominent positions in curriculum development and teacher training."

At the university level, mathematics educators often learn from one another. Chinese mathematicians, for example, faced with the projected large increase in the number of their university students, have reached out to American mathematicians to learn how we changed the teaching of calculus when university enrollments in the US almost tripled between 1965 and 1975 (L. Friedler, 2004.) In the other direction, American researchers in mathematics education have been influenced by the Chinese approach to the teaching of elementary school mathematics articulated by Liping Ma (1999).

There are still important differences, nevertheless, in Eastern and Western approaches to mathematics and mathematics curricula. In China and Hong Kong, curricula tend to stress computational skill more than Western curricula do, and this emphasis may partially explain recent results on cross-national studies. (See section 3D below.) Wong, Han, and Lee point out (2004 p. 62) that “Eastern curricula are more centralized and focused more on content.” These authors stress the “Product versus Process” dichotomy between Eastern and Western curricular trends. In spite of these global differences, the president of the African Mathematical Union is quoted in Atweh and Clarkson as “warning against the over-emphasis on culturally oriented curricula for developing countries that act against their ability to progress and compete in an increasingly globalized world.” (2002A p. 10).

Our conclusion is that there is indeed an increasing uniformity of mathematics curricula worldwide. We want our students to be aware, however, that there are still substantial cultural differences in approaches to mathematics.

C. Global collaboration

One of our objectives is: “Students will have first-hand experience interacting with creators and users of mathematics from other countries and cultures”. This is particularly relevant given that research in mathematics is increasingly practiced both internationally and collaboratively. A recent preliminary report from New Mexico State University for the American Council of Education (unpublished) asked each department to report its internationalization efforts. Most departments at their university reported that 20-50% of its faculty was involved in international efforts. Yet 100% of New Mexico State’s math faculty reported such involvement. The data simply reflect the fact that almost all research in mathematics is published in English, is read by mathematicians around the globe, and increasingly involves international collaboration. For example, a recent study in Canada (Bond & Lemasson, 2000) found that of all disciplines, mathematics has the highest degree of international collaboration (between Canadian authors and others outside of Canada). This collaborative experience is central to mathematics. Many more math-related papers are co-authored today than 30 to 40 years ago. J.W. Grossman (Grossman, 2002) has used data from Mathematical Reviews to conclude that “in the 1940’s and 1950’s nearly 90% of all papers were solo works, while in the late 1990’s fewer than half of all papers had just one author.” It is necessary to be aware of cultural differences in order to have successful international collaboration. For example, the second author experienced a partial collapse of a potential project with mathematicians from another country because early in the process he missed cultural cues about the direction they preferred. As mathematics educators, it is imperative for us to determine how to encourage our students to learn more of other cultures so that they will be able to be successful mathematicians. Study abroad is a very effective way of achieving first-hand awareness of cultural differences. Unfortunately, math and science students participate in study abroad programs at a lower frequency than other majors. Less than

2% of all US students studying abroad in 2005 were mathematics or computer science majors (Open Doors, 2006). Further, it follows from this data that less than 1% of all mathematics majors in the US study abroad. The data from Europe is only marginally better. The most recent survey from Erasmus (Erasmus, 2006) shows that 3.4% of all students who study within another EU country are mathematics or computer science majors. (The percentage of EU mathematics majors who study abroad does not clearly follow from this published data.) The percentage of non-US students who study in the US is very similar: 2% of international students who study in the US are mathematics majors (Open Doors, 2006). By comparison, 6.1% of these international students in the U.S. are computer science majors and 15.7% are engineering students. We need to encourage more university mathematics students to study abroad.

D. Tests comparing students of different countries

In the last 20 years there have been many studies comparing the mathematical performance of elementary and secondary students of different countries. Our students should be aware of two debates surrounding comparison testing in mathematics. The first one questions the notion that Western-based tests taken by students of different countries can truly rank and evaluate the quality of the mathematics teaching in these countries. The second suggests that tests results alone give misleading conclusions; it is necessary to analyze the types of problems in which certain countries excel.

Robitaille and Travers (1992) suggest that a primary reason to participate in these tests is so that "different approaches to the same goal can be compared." To participate is, of course, to assume a shared and accepted sense of the goals involved; recent critiques of the international PISA test raise the question of whether or not this is a reasonable assumption (Clarke, Keitel & Shimizu, 2006). As to why mathematics is the subject so often compared, Robitaille and Travers give three reasons: math plays a prominent role in the curriculum of every country; there is a similarity of content in mathematics curricula internationally; the language of mathematics is international. However, if the mathematical traditions are different, what is to be learned? Atweh and Carlson (2002A) discuss the example of Brazil, which has refused to participate in a number of international achievement studies, choosing instead to evaluate their education internally. Atweh and Carlson quote a participant in a focus group on the globalization of mathematics who notes that "these evaluations are promoted by the World Bank and by IMF, so there is a clear interest in this financial system to have a good mathematics education. And they are against, of course, ethnomathematics. They want good international standards." Thus, the question of whether or not to test internationally is closely related to the question of internationalization of curricula and to the place of ethnomathematics, issues that are discussed in sections 3A and 3B above.

In 2000, an international test, the Trends in International Mathematics and Science Study (TIMSS), showed "American students, over all, performing worse in math and science than students in Singapore, Taiwan, Russia, Canada, Finland, Hungary, the Netherlands and Australia." (Schemo, 2000) These test results led to a wide-spread belief that American mathematical education does not measure up to international standards. We may be drawing conclusions too quickly, however; recent literature seems to indicate that the actual relationship between American and other international mathematics education is more complex. For example, a series of cross-national studies by Jinfa Cai and his students at the University of Delaware (Cai, 1995; Cai & Cifarelli, 2004) suggest that Chinese elementary and secondary students far outperform American students in computational mathematics but are marginally behind in conceptual problem solving. Similar results were found by Judson and Nishimori comparing US and Japanese high school students in calculus (2005). Jun Chai's preliminary results of administering the AP Calculus exam to Chinese university freshmen showed the same trends (Chai, 2005). Perhaps these results point to the need for a US curriculum that blends what it does best-teaching concepts, with the computation necessary to compete internationally. This is exactly the conclusion of the latest NCTM report: Curriculum Focal Points (2006).

Section 4: Our Proposal

In this section, we make suggestions for changing the university mathematics curriculum to meet our learning objectives. These revisions acknowledge both the international nature of the practice of mathematics today and the importance of cultural context. Our approach is influenced by the general themes of Section 3, the authors' mathematical backgrounds, and the general education requirements of our own university. We realize that most mathematics departments could not adopt all of these ideas; we are simply putting forth some possibilities.

A. New course: Cultural Aspects of Mathematics. We propose a new course to be team-taught by the Departments of Mathematics, Sociology/Anthropology and (perhaps) Education. This new class will be open to all students as a general education course. We envision a course that will have prerequisites of either a sophomore level mathematics course or a sophomore level sociology course, so that all students will bring background knowledge in one of these areas. We believe this will create a natural forum in which to discuss the issues raised in Section 3 as well as other topics related to culture and mathematics. This course is designed to meet our Objectives 1 and 2, and more specifically the outcomes 1b, 1c, 2a and 2b. Topics in the course may include the following.

> a. **Are Asian students more capable than American students in mathematics?** Students will examine recent test results and scholarship and will investigate the question of whether performance on mathematics exams is affected by cultural expectations or the history of mathematics in a country or culture. Students will also consider the effect of the increasingly international nature of elementary-secondary mathematics curricula on world-wide mathematics education. These issues will lead to a broader discussion of whether there exists or should exist a cultural approach to mathematics.
>
> **b. Can mathematics be separated from cultural context?** Students can learn, for example, about an interdisciplinary group based at MIT that studies the khipu knots of the Inca Empire, which were used to encode and decode information about the workings of their society. The khipu is a system of knotted ropes that are attached

to a longer cord. The aim of the Khipu Research Group is to break the khipu code. Although the work of this group is at a graduate level, it will be used to illustrate two key features of our approach. The first one is that the mathematical concepts used in this project are sophisticated and part of the mainstream mathematics of today. The second is that there is no hope of making progress in this project if one reduces it to a pure mathematical problem. In fact, it is arguable that there can be no such reduction since the knots are a method of record keeping for matters of significance to that culture. The cultural component of the problem is indissolubly tied to its mathematical analysis.

c. What is the role of mathematics in different cultures? Students could investigate the history of Cartesian coordinate systems and their uses in a variety of mathematical models, including their extension to the study of linear systems and spaces, and observe how their use in Western society contrasts with other cultures such as the Navajo (Pinxten, VanDoren, & Soberton, 1987) and the Inuit (Poirier, 2004), in which Cartesian systems and modes of perception are not the norm. Does the concept of geometrical distance (or lack thereof) tell us anything significant about a culture? Another question to consider might be the relationship between the Romans' lack of a zero and their ability to abstract.

d. How does mathematics influence controversial social issues? Students will examine the consequences of mathematical modeling used for racial profiling, income distributions, allocation of resources, welfare distribution algorithms, and other applications of mathematics to social decision-making (Gutstein & Petersen, 2005; Skovsmose, 2006). The emphasis will be on the social effects of mathematical modeling rather than on the modeling itself. Students will study the effects of reducing complicated interactions to a usable set of independent and dependent variables and the potential distancing of social objectives such as equity and justice from the eventual application of algorithms that strive to achieve these objectives. Another topic for discussion could be the occasionally inappropriate uses of mathematical modeling for situations that would be better served by other intellectual tools. Mathematics in these contexts serves a formatting function (Skovsmose, 2006; Keitel et al., 1993) that constrains and enables the field of possible thoughts and actions in specific ways. For example, students might be asked to create a usable algorithm for equitable distribution of welfare benefits, and to critique their algorithms; or they might study the uses of mathematical arguments in gender equity court cases that have resulted in different juridical outcomes.

e. Mathematics and the West.

In this module of the course, students will consider two questions: Is mathematics dominated by the West? Is there a recognizably Western mathematics? The related question of whether there is a Western approach to mathematics will be studied in the module 4Aa ("Are Asian students better at math?") above and also in the revised history course (section 4C).

The first question will be approached by examining the globalization of curricular development (section 3B), the place of ethnomathematics in the curriculum (section 3A), and whether the contemporary canon excludes, intentionally or not, contributions from non-Western cultures. For example, both khipu knots (see section 4Ab) and Penrose tiling (see section 4C below) demonstrate that some non-Western civilizations developed sophisticated mathematics that is only now becoming known to Western mathematicians.

The second question, “Is there a recognizably Western mathematics?”, will be studied partially through the writings of mathematicians who believe that mathematics is universal. Students will contrast this opinion with the work of Bishop and Nunes (Bishop, 1991; Nunes, 1992), who, as pointed out in Section 3A, have suggested that the same set of questions arise independently in different cultures. Students will also consider the question (Walkerdine, 1987) of whether mathematical thinking and reasoning have become models of rationality in Western society. If so, then they will examine the role that mathematics plays in the creation of public policies.

B. Revised Mathematical Modeling course. To satisfy learning objective 4 (“Students should be able to view the role that mathematics plays in modeling and solving problems from an international perspective”), we propose a new mathematical modeling course. Many university mathematics departments already offer an upper-level course in mathematical modeling. These courses typically introduce stochastic or deterministic models, often using techniques of dynamical systems, probability, and graph theory. We propose expanding that approach in two directions. First, it would be quite natural to include the modeling of international problems such as global warming, weather- induced crises, economic interactions, the designing of low-cost shelters and transportation, environmental impacts, etc. These problems would lead naturally to a discussion of our second addition to the standard course: understanding the limits of mathematical modeling. Although there might be, for example, a concise mathematical solution to a problem, the resources to implement that solution might be limited; the optimal solution which can realistically be implemented might not be the mathematically best solution.

Indeed, there have been several papers in the last few years modeling catastrophic events such as earthquakes, hurricanes, and terrorist attacks (e.g., Sorette, 2002). These papers discuss the inherent limitations of modeling; in Sorette’s example of large scale non-linear models, there are mathematical limits to predicting events because of chaotic behavior. Equally relevant, however, is the distribution of resources after a loss. For example, does the U.S. Department of Homeland Security allocate anti-terrorism resources on the basis of a prediction of the actual risk, or are there also political factors which influence a decision? (Lipton, 2006). Similarly one might ask how resources were distributed after Hurricane Katrina or after the tsunami in Asia.

For another example, graph and network models, called “small world networks,” are increasingly used by social scientists to study group interaction. Researchers are aware of the “limitations of the models due to the social structure that is not captured by the graph.” (Newman, Wallen, Strogatz, 2002); a study of the graph model is not useful without understanding the social structure it is modeling.

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C. **Revised Math History course**. As mentioned in Section 3A, Asher (2003) and others have called for an infusion of non-Western ideas into the standard mathematics history course. For instance, there is very little understanding in the West that China’s mathematical development was much more centered on computational/algorithmic methods for solving problems, and this tradition affects how the Chinese approach mathematics today. This background might also partially explain the results of cross-national studies of Asian and American students discussed in Section 2 of this paper. Chinese history may also affect directions of current research. For example, Martzloff, in his *History of Chinese Mathematics* (Martzloff, 1997) comments that “Wu Wenjun, a contemporary Chinese mathematician, views his research on algorithms and automatic theorem proving as a continuation of the ancient tradition of Chinese mathematics.” Such an ‘infused’ course could also include discussion of the role of mathematics in different cultures. Again using China as an example, the civil service exams that were developed in the late Han Dynasty (202 B.C.E.-220 C.E.) required knowledge of arithmetic; that is, mathematics was central to the basic knowledge required of educated people. This course is designed to fulfill Outcome 1a.

A less traditional, but potentially more interesting, introduction to mathematics history would be to change from a Western chronological approach to one whose organizing principle is mathematical concepts through time. One advantage of this reorganization would be to avoid telling a simple story of “progress” and thus would be more in keeping with modern approaches to history. J. Gernet, in the forward to Martzloff’s *History of Chinese Mathematics* (1997, p vii) comments, “Although our mathematics has now become the common heritage of humanity, our understanding of mathematics is essentially based on a tradition peculiar to ourselves which dates back to ancient Greece; in other words, it is not universal.” In our proposed course, there could be a unit on geometry, including Egyptian and Babylonian geometry, Greek emphasis on axiomatic proof, the “method of exhaustion” to find areas used by Archimedes and Chinese mathematicians of the same period, discussion that China never developed an axiomatic geometry, non-Euclidean geometries, and the recent discovery that “quasi-crystalline Penrose designs were discovered by Islamic artisans” 500 years before Western mathematicians (Wilford, 2007). Using this conceptual approach or the modified traditional one outlined above, it would still be necessary to comment critically on the mathematics studied. For example, symmetry arises frequently in history; these examples are interesting, but it is unlikely that the designers recognized the abstraction of symmetry groups in their designs. On the other hand, constructing the Penrose tilings described above certainly reflects sophisticated reasoning that the West has only recently discovered.

Other disciplines have successfully used a conceptual approach to the history of their subject. For example, at Swarthmore College the traditional history course in Western Dance has been remade as World Dance Forms, in which concepts such as “use of space” or “inclusion of music” are studied in different cultures at different times. (S. Friedler, 2007). Applying this approach to mathematics will clearly require a specialist in the history of mathematics.

D. **Study abroad.** We propose an increased emphasis on study abroad for mathematics majors, for the reasons discussed in Section 3C. As documented in that section, fewer mathematics students participate in this experience than those in the humanities and social sciences although they are more likely to be international collaborators later in their careers. This emphasis is designed to satisfy Outcome 1b.

A department's ability to encourage mathematics majors to study abroad is highly dependent on the individual university and its students. At Arcadia University, faculty have recently adopted a new general education curriculum that contains the following statement: "Students are required to have a sustained cross-cultural experience that places them in cultural settings different from those in which they have previously lived." (Arcadia University, 2007) In addition, Arcadia also continues its London/ Scotland/ Madrid Preview. First year students are offered the opportunity to travel to one of the three sites for a week of education and sight-seeing. The cost to students is minimal (currently USD 250) and is underwritten by our university. It is our hope that the experience of international travel, encouragement from our faculty, and the new requirement will lead to increased international study for our mathematics majors.

E. Revised Mathematics Capstone Course. Many university mathematics departments require their senior mathematics majors to have a capstone experience or course. Students typically write a paper containing either original work or library research. Often mathematics departments have their students enroll in a formal course that includes such components as discussions of careers in mathematics and talks from mathematicians. In the case of Arcadia University, we currently require students to write a paper as a final project in one of several courses. We propose to develop a course which will be similar to the traditional courses but will have approximately one quarter of its content devoted to cultural issues similar to those in the proposed Cultural Aspects of Mathematics course. Students will learn to place their mathematics in the context of Western mathematics traditions and to understand how that might differ in a non-Western setting. We feel that all mathematics majors should be exposed to these ideas and since the interdisciplinary course will be an elective, we will include similar material here. For those universities which do not develop a stand-alone interdisciplinary course, the capstone is an ideal place to introduce these issues.

Section 4: Conclusions

In this paper we have presented learning outcomes, discussed issues related to internationalizing the university mathematics curriculum, and offered specific suggestions about how to accomplish these goals. We realize that no department could undertake all of these suggestions. In the case of the authors, we will begin by developing the Cultural Aspects of Mathematics course at Arcadia University. Soon thereafter, we expect to modify our existing History of Mathematics course by infusing it with non-Western examples and approaches. Other suggestions will take us longer to implement. Our long-term goal is to infuse all courses with an international perspective consistent with our learning objectives. Meanwhile, we plan to continue our discussions as a group, with the intent of further developing our ideas. We also hope to interact with others in the mathematics and mathematics education communities who share our interests and concerns.

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